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Performance of Adaptive Matched Filter Receivers Over Fading Multipath Channels

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Abstract—Various types of noise inherent in the channel estimation process for a fading multipath channel are examined and modeled. Performance degradation due to noisy channel estimation in the adaptive matched filter receiver is analyzed, and some numerical results—contrasting reference-directed and decision-directed channel estimation—are presented for the troposcatter channel.

I. INTRODUCTION

SINCE the mid-1950's different types of adaptive receivers have been studied for a variety of fading multipath channels. Historically, the most important adaptive receiver for the fading multipath channels is the RAKE system [1]. The RAKE system was originally designed for teletype with a baud rate of 90 cps working over an ionospheric channel. The envelope of the transmitted binary FSK signal is a 10 kHz PN-sequence. The received signal is passed through two tapped delay lines—the matched filters—for mark and space frequencies, and the outputs are compared for decision making. The tap gains of the delay lines are adaptively adjusted by cross correlating the received signal with both mark and space reference signals at the receiver. Since the data rate is considerably smaller than the coherent bandwidth of the channel, the intersymbol interference (ISI) is negligible. More recently, other versions of RAKE receivers have been examined for urban radio [2], HF radio [3], and indoor radio [4] channels.

In the past two decades, other adaptive design approaches have emerged in the development of a new generation of radios for fading multipath channels. Time gating of the transmitted pulse to avoid intersymbol interference (ISI) and adaptive matched filtering on each received pulse was the approach taken for a family of military troposcatter radios [5], [6]. The adaptive decision feedback equalizer was another approach examined for radio receivers for troposcatter [7], [8], microwave line-of-sight [9], [10], HF [11], and more recently indoor radio [12]. Finally, an adaptive version of the maximum likelihood sequence estimation (MLSE) [13] was considered for troposcatter in [14] and HF in [15].

This paper examines the effectiveness of three different techniques for the implementation of an adaptive discrete matched filter receiver operating over a fading multipath channel. The analysis is for the slow-fading wide sense stationary uncorrelated scattering (WS-SUS) channel [16], and the numerical results are based on measured characteristics of a troposcatter channel. Typical systems which can directly use the results of this analysis for performance predictions

are time gated adaptive matched filters [5], [6] and the spread spectrum adaptive RAKE receivers [3], [16]. For other receivers such as decision feedback equalizers (DFE) or maximum likelihood sequence estimators (MLSE), the resulting performance predictions represent an ideal limit to which practical systems can aim.

Any implementation of an adaptive matched filter receiver for fading multipath channels requires a reference signal. It can be realized by transmission of a known probe signal (transmit reference) or by removing the data modulation (decision directed) from the received signal. The probe signal can be either added directly to the transmitted signal as a low power spread-spectrum signal (parallel probe) or it may be time multiplexed with the transmitted symbols (serial probe) [3], [5], [16]. Either realization has shortcomings which result in noisy estimates of the channel. One source of this noise is the data themselves. With the decision-directed method, the reference signal is derived from detected bits which are presumed correct. Data detection errors result in an incorrect reference. With the transmit reference approach, the reference is separated from the total received signal, and the mere presence of the data signal in the channel estimation circuitry is a source of noise. Another source of noise is tracking error: a finite amount of time is needed to measure an ever-changing channel impulse response. A final source of noise is the thermal noise present in the received signal.

For an adaptive matched filter receiver with negligible ISI, such as the time gated adaptive matched filter or a RAKE system, the effects of measurement noises are analyzed. Moreover, probing versus decision-directed adaptation is compared with respect to the system performance where the criterion is average probability of error. This will be done using some computer generated numerical results. Section II provides a general discrete model for the impulse response of the fading multipath channels. Section III considers both theoretical and implementational issues regarding measurement of the discrete channel impulse response. Section IV introduces different sources of noise which are involved in reference-directed and decision-directed channel measurement techniques. Section V analyzes the performance of a discrete matched filter receiver with noisy channel measurements; and Section VI provides the numerical results and discussions.

II. A GENERAL DISCRETE MODEL FOR FADING MULTIPATH CHANNELS

The baseband model considered is shown in Fig. 1, and it is a general model of a high-speed digital communication system over a fading multipath channel. In this section, the model, from the information source at the transmitter up to the input of the digital processor in the receiver, will be recast as a discrete tapped-delay line. Use will be made of the assumption that the transmitted signal is band-limited.

The data sequence a_n modulates an impulse train $\delta(t - nT)$ where T is the symbol period. The impulse train acts as an input to a filter whose impulse response, $f(t)$, is the fundamental transmit-

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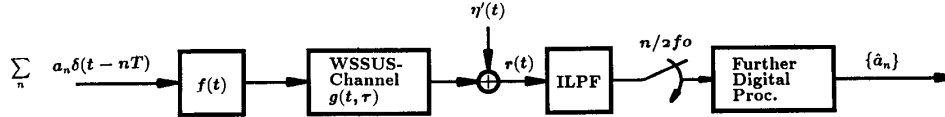


Fig. 1. Overall baseband model for communication over wide sense stationary uncorrelated scattering (WSSUS) fading multipath channels.

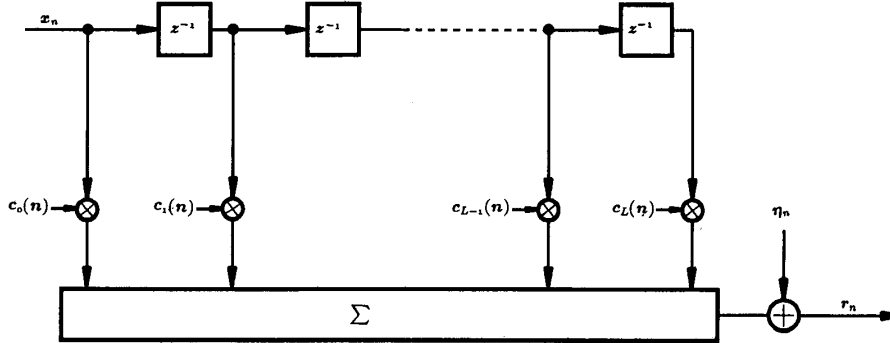


Fig. 2. The discrete model for the channel which represents the transmitter filtering, the WSSUS channel, additive noise, and the prefiltering at the receiver.

ted symbol; the output of the filter,

$$\sum_k a_k f(t - kT),$$

is the transmitted signal. It is assumed that $f(t)$ is band-limited to f_0 Hz.

The signal passes through a fading multipath channel with time-variant impulse response $g(t, \xi)$. The function $g(t, \xi)$ represents the path gain associated with delay ξ at time t . White, complex Gaussian noise of two-sided spectral height $N_0/2$ is added to the information bearing signal to form the received signal

$$\sum_k a_k c(t, t - kT) + \eta'(t) \quad (1)$$

where

$$c(t, u) = \int g(t, \xi) f(u - \xi) d\xi.$$

The receiver would normally perform pulse shaping filtering followed by a sampler at the output, operating at the symbol rate of $1/T$ samples/s. This is an optimal way to process $r(t)$ in the absence of multipath. But since $f(t)$ and hence $c(t)$ are band-limited, it is also optimal to pass $r(t)$ through an ideal low-pass filter followed by a sampler operating at the Nyquist rate. In this manner, all critical processing is deferred to a subsequent digital processor.

The sampled output is

$$r(t)|_{t=n/2f_0} = \sum_k a_k c(t, t - kT)|_{t=n/2f_0} + \eta(t)|_{t=n/2f_0}$$

or

$$r_n = \sum_k a_k c(t, t - kT)|_{t=n/2f_0} + \eta_n, \quad (2)$$

where $\eta(t)$ is the response of the ideal low-pass filter (ILPF) to $\eta'(t)$. The noise samples are zero-mean, independent, complex Gaussian random variables with variance

$$E\{|\eta_n|^2\} = 2N_0f_0 = N_0.$$

Equation (2) is not in a convenient form since it represents sampling at one rate and transmission at another rate. Define

$$M = 2f_0T,$$

and adjust f_0 larger, if necessary, to ensure M is integer. Define the discrete sequence

$$x_n = \begin{cases} a_{n/M}; & \text{for } n/M \text{ integer} \\ 0; & \text{otherwise} \end{cases} \quad (3a)$$

which consists of the data interleaved with $(M - 1)$ zeros. Then the sampled output may be written compactly as

$$r_n = \sum_k x_k c_{n-k}(n) + \eta_n = \sum_k x_{n-k} c_k(n) + \eta_n \quad (4a)$$

where

$$c_k(n) = c(t, u)|_{t=n/2f_0, u=k/2f_0}.$$

Since $c(t, u)$ is band-limited, it cannot also be time-limited. However, in all practical situations, $c(t, u)$ will have a finite duration by some suitable engineering definition. Let $(L + 1)$ denote the number of samples in that duration.¹ Then the model corresponding to (4a) is shown in Fig. 2 where sufficient delay is added to make it causal.

For the negligible ISI, $M \geq L + 1$, there is a vector of sampled received sequences with length $L + 1$ corresponding to each single symbol a_n . Therefore, (4a) reduces to

$$r_{n-k} = a_n c_k(n) + \eta_{n-k}; \quad 0 \leq k \leq L \quad (4b)$$

at the baud rate.² In a vector form, this equation is written as $\mathbf{R} = a_n \mathbf{C} + \boldsymbol{\eta}$. For each transmitted symbol, a vector of length $L + 1$ is received, resulting in $L + 1$ order of implicit diversity. To normalize (4a) and (4b), throughout the analysis it is assumed that $a_n = \pm \sqrt{E_b}$ where E_b is the transmitted energy per bit, and

$$\sum_{k=0}^L E\{|c_k(n)|^2\} = 1.$$

¹Since $E\{|c_k(n)|^2\}$ is the average power associated with the tap weight, one method of choosing L is to retain all taps whose power is within so many decibels of the largest tap power. The criterion of 10 dB worked well for the numerical results.

²Note that during transmission time of several symbols, the channel is assumed to be fixed, i.e., $c_k(n) = c_k(n + k)$ for small values of k .

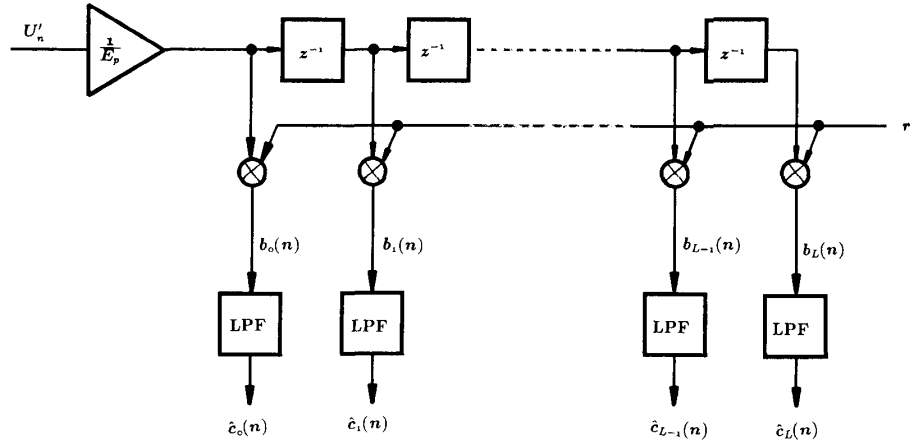


Fig. 3. Discrete channel estimation using cross correlation.

A. WSSUS Model for the Channel

The channel impulse response, $g(t, \xi)$, is assumed to be a WSSUS zero-mean white Gaussian process. The correlation function for this channel is given by [15]

$$E\{g(t, \xi)g^*(t - \tau, \lambda)\} = Q(\tau, \lambda)\delta(\lambda - \xi).$$

The term

$$Q(0, \lambda) = Q(\lambda)$$

is known as the "delay power spectrum" [15], and it reflects the power density at different multipath delays. The numerical results of this paper use a delay power spectrum of

$$Q(\lambda) = \begin{cases} b^2 \lambda e^{-b\lambda}; & \text{for } \lambda \geq 0 \\ 0; & \text{for } \lambda < 0 \end{cases}$$

where b is related to the rms multipath spread of the channel, δ , through the expression

$$b = 2 \frac{\sqrt{2}}{\delta}.$$

This function is an approximate curve fit to typical measurements for troposcatter links [17], which is consistent with the theoretical predictions in [15].

III. CHANNEL MEASUREMENT TECHNIQUES

For the adaptive matched filter analyzed in this paper, the digital processor in Fig. 1 performs two adaptive operations. One is the measurement of the instantaneous overall channel impulse response, the other is adaptive matched filtering. In this section we explore different techniques for the instantaneous channel measurement.

The reference signal will be denoted by $\{u_n\}$. Without any loss of generality, we perform the rest of our calculations for the inphase channel component by assuming data $a_n = \pm \sqrt{E_b}$ and probe $u_n = \pm \sqrt{E_p}$. Since the sum of average received power for all tap gains is normalized to one, E_b and E_p represent the energy per data bit and energy per probe bit, respectively. For the transmit reference approach, $\{u_n\}$ is the transmitted probe sequence; and for the decision-directed approach, $u_n = \hat{a}_n$ where $\{\hat{a}_n\}$ is the detected symbol. Similar to (3a), one may define the probe signal at the sampling rate as

$$u'_n = \begin{cases} u_{n/M}; & \text{for } n/M \text{ integer} \\ 0; & \text{otherwise.} \end{cases} \quad (3b)$$

A. The MMSE Solution for Channel Estimation

The input to the channel estimator is given by

$$r_n = \sum_{k=0}^L c_k(n) u'_{n-k} + \eta_n \quad (5)$$

where $c_k(n)$ is the equivalent discrete channel impulse response, u'_n is the reference signal, and η_n is the additive noise. The channel estimator forms the estimates of the channel impulse response $\hat{c}_k(n)$ and the received signal

$$\hat{r}_n = \sum_{k=0}^L \hat{c}_k(n) u'_{n-k}.$$

It is well known [16], [18] that the MMSE estimate of the channel impulse response which minimizes the $E\{|r_n - \hat{r}_n|^2\}$ satisfies the following linear set of simultaneous equations:

$$AC = B \quad (6a)$$

where C is the vector of MMSE estimate of the tap gains, and the matrix A and the vector B are defined by

$$\begin{aligned} a_{ij} &= E\{u'_{n-j} u'_{n-i}\}; & 0 \leq i, j \leq L \\ b_i &= E\{r_n u'_{n-i}\}; & 0 \leq i \leq L. \end{aligned} \quad (6b)$$

For a statistically independent reference signal, such as a PN-sequence, A is a diagonal matrix with E_p the value of all diagonal elements. The solution to (6a) is then

$$\hat{c}_k(n) = \frac{1}{E_p} E\{u'_{n-k} r_n\} \quad 0 \leq k \leq L. \quad (7)$$

B. Implementation, Cross Correlator, and LMS Algorithm

There are two ways to implement the channel estimator discussed in the last subsection. One is to design a cross correlation to calculate (7) directly. The second is to use the equalizer-like channel estimator described in [19]. This estimator is implemented by applying the steepest descent algorithm for minimization of MSE, which was first introduced and named the least-mean-square (LMS) algorithm by Widrow [18].

Fig. 3 represents the implementation for the cross correlator. The reference signal is passed through a tapped-delay-line (TDL), the tapped signals are multiplied with the received signal and then passed through a bank of low-pass filters. To normalized the output of the filters to actual samples of the channel impulse response, one of the signals is scaled to $1/E_p$ [see (7)].

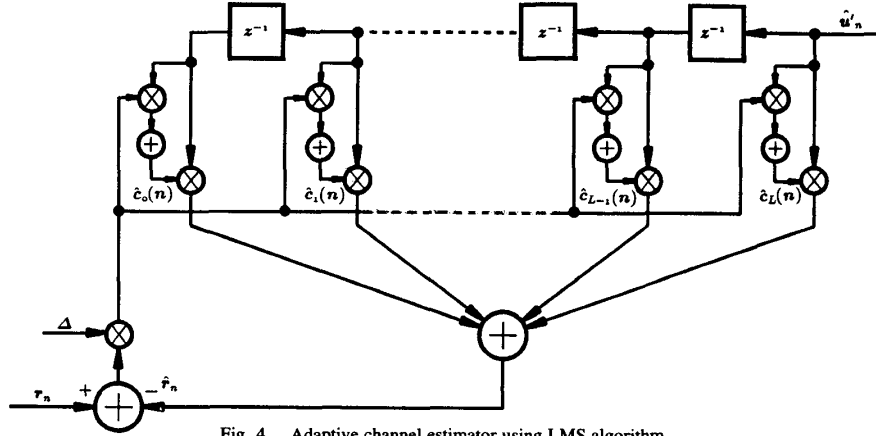


Fig. 4. Adaptive channel estimator using LMS algorithm.

Fig. 4 represents the equalizer-like channel estimator using the LMS algorithm. The reference signal is passed through a TDL and the channel tap gains are adjusted according to

$$\hat{c}_j(n+1) = \hat{c}_j(n) + \Delta e_n u'_{n-j} \quad (8)$$

where Δ is the adjustment rate parameter and

$$e_n = r_n - \hat{r}_n = r_n - \sum_{k=0}^L \hat{c}_k(n) u'_{n-k}. \quad (9)$$

Substituting (9) in (8) and assuming $u'_n = \pm \sqrt{E_p}$, we have

$$\begin{aligned} \hat{c}_j(n+1) = & \hat{c}_j(n) [1 - \Delta E_p] \\ & + \Delta \left[r_n u'_{n-j} - \sum_{\substack{k=0 \\ k \neq j}}^L \hat{c}_k(n) u'_{n-k} u'_{n-j} \right]. \end{aligned} \quad (10)$$

Note that $|u'_{n-j}|^2 = E_p$.

Considering (3b) with $M \geq L+1$, the last term on the right-hand side is zero. The remainder of (10) shows a first-order difference equation. It can be implemented by passing $r_n u'_{n-j}$ through a bank of first-order digital filters with system function

$$H(z) = \frac{\Delta}{z - (1 - \Delta E_p)}. \quad (11)$$

This is similar to the multiplication and low-pass filtering operation used for cross correlation. As a result, the LMS channel estimator is equivalent to the cross correlator in Fig. 3. In both cases, the received signal r_n is multiplied with the delayed reference signal u'_{n-j} and the result is low-pass filtered. In the LMS estimator, the low-pass filter is restricted to a first-order filter defined by the system function given by (11). For the cross correlator, however, the low-pass filter can be designed with any order.

IV. MEASUREMENT NOISES

There are three noise sources affecting the channel measurement process: thermal noise, inherent to any communication system; data noise caused by the structure of the channel estimator and the transmitted digits; and tap gain variation noise generated because of tracking errors in estimating a time-varying process.

In Fig. 3, the expected values of both input and output of the low-pass filters are the value of the tap gain: $E\{b_k(n)\} = E\{\hat{c}_k(n)\} = c_k(n)$. Therefore, it can be assumed that if $b_k(n) = c_k(n) + v_{nk}$ then $\hat{c}_k(n) = c_k(n) + w_k$, where v_{nk} and w_k are the measurement noise before and after cross-correlator filtering. Assuming all measurement noises are white Gaussian, the relation between variance

of w_k and v_{nk} is given by

$$\begin{aligned} \sigma_w^2 &= E\{|\hat{c}_k(n) - c_k(n)|^2\} \\ &= \frac{\sigma_v^2}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega})|^2 d\omega \end{aligned}$$

where $H(e^{j\omega})$ is the discrete time frequency response of the low-pass filters. The numerical value of the integral in the above equation is proportional to the bandwidth of the low-pass measuring filter

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega})|^2 d\omega = C_1 B_D$$

where B_D is the bandwidth of the filter in discrete frequency (Hz/samples) and C_1 is a constant. This filter is operating at the rate of arrival of the probe sequence. Assuming one probe digit is sent for each α digits of information, the average sampling rate used for the filter input is $\frac{1}{(\alpha+1)T} \approx \frac{1}{\alpha T}$ where T is the baud interval. Then, the physical bandwidth B_f of the filter in Hertz is $B_f = \frac{B_D}{\alpha T}$.

Defining normalized bandwidth, B_N , as the ratio of the bandwidth of the filter in Hz to rms Doppler spread $B_N = \frac{B_f}{\sigma_D}$, σ_w^2 can be written as

$$\sigma_w^2 = \sigma_v^2 C_1 B_D = (C_1 \alpha T) \sigma_D^2 B_N. \quad (12)$$

The Doppler spread reflects the rate of variation of the channel impulse response. The bandwidth of the low-pass filter used for channel estimation determines the tracking speed of the measurements: the wider the bandwidth, the faster the tracking. To keep the channel variations during the measurement time negligible, the bandwidth of the measurement filter should be several times larger than the Doppler spread of the channel. Therefore, the normalized bandwidth is usually much greater than one.

Equation (12) relates measurement noise to normalized bandwidth of the measuring filters B_N , type of measurement filters C_1 , rms Doppler spread of the channel σ_D , baud interval T , format of the reference signal α , and variance of the measurement noise at the input of the measurement filter σ_v^2 . In the rest of this section, we define various measurement impairments and we relate them to σ_v^2 . Various components of σ_v^2 are due to thermal noise, σ_T^2 , tap gain variation noise, $\sigma_{\Delta v}^2$, and structural or internal noise, σ_I^2 .

A. Variance of Thermal Noise σ_T^2

If additive thermal noise η_n with variance σ_η^2 is the only source

of impairment, then from (5), (4b), and Fig. 3 we have

$$\sigma_T^2 = \text{Var} \{b_k(n)\} = E \left\{ \left| \frac{1}{E_p} r_{n-k} u_n - c_k(n) \right|^2 \right\} = \frac{\sigma_\eta^2}{E_p}. \quad (13)$$

Note that $E\{|u_n|^2\} = E_p$, and during the measurement time $E\{b_k(n)\} = c_k(n)$.

B. Variance of Tap Gain Variation Noise $\sigma_{\Delta v}^2$

The measurement noise was found under the assumption of no tap gain variations during the measurement time. However, tap gains are samples of a complex Gaussian low-pass process with the bandwidth of σ_D , and they change slightly during the measurement time τ_F . For the estimation of the channel tap gain at time t , the tap gain values between t and $t - \tau_F$ are used for the measurements. On the average, these values are $\tau_F/2$ apart from the desired value at t . Variance of the tap gain variations for the average delay between the samples used for measurement is given by

$$\begin{aligned} \sigma_{\Delta v}^2 &= E \left\{ \left| c_k(t) - c_k \left(t + \frac{\tau_F}{2} \right) \right|^2 \right\} = 2 \left[R_c(0) - R_c \left(\frac{\tau_F}{2} \right) \right] \\ &\approx \frac{2}{3} \pi^2 \sigma_D^3 \tau_F^2 \end{aligned}$$

where $R_c(t) = \frac{\sin 2\pi\sigma_D t}{\pi t}$ is the autocorrelation function of the tap gains in time domain.

The measurement time is inversely proportional to the bandwidth of the low-pass filter used for measurement

$$\tau_F = \frac{C_2}{B_f} = \frac{C_2}{B_N \sigma_D},$$

in which C_2 is a constant. Therefore,

$$\sigma_{\Delta v}^2 = \left(\frac{2}{3} \pi^2 C_2^2 \right) \frac{\sigma_D}{B_N^2}. \quad (14)$$

C. Variance of Internal Measurement Noise σ_I^2

When the probe sequence is superimposed onto the data digits as a spread spectrum probing signal, the received sampled signal at the baud rate is given by

$$r_{n-k} = (a_n + u_n) c_k(n) + \eta_{n-k}; \quad 0 \leq k \leq L.$$

The input to the low-pass filters is

$$\frac{1}{E_p} \{r_{n-k} u_n\} = \frac{|u_n|^2}{E_p} c_k(n) + \frac{u_n a_n}{E_p} c_k(n) + \frac{u_n}{E_p} \eta_{n-k}.$$

The third term on the right-hand side of the above equation is the thermal noise. The second term, $\frac{u_n a_n}{E_p} c_k(n)$, is another source of measurement noise caused by the cross-talk between probe digits and information digits. We refer to this noise as the internal measurement noise for parallel probe. The variance of the noise σ_I^2 is

$$\begin{aligned} \sigma_I^2 &= E \left\{ \left| \frac{u_n a_n}{E_p} c_k(n) \right|^2 \right\} \\ &= \frac{E_b}{E_p} E \{ |c_k(n)|^2 \} \\ &= \frac{E_b}{E_p} \beta_k, \end{aligned} \quad (15)$$

TABLE I
MEASUREMENT NOISES

$\sigma_v^2 = \sigma_T^2 + \sigma_{\Delta v}^2 + \sigma_I^2$	Thermal σ_T^2	Tap Gain $\sigma_{\Delta v}^2$	Internal σ_I^2
Serial Probe.	$\frac{N_0}{E_p}$	$\left(\frac{2}{3} \pi^2 C_2^2 \right) \frac{\sigma_D}{B_N^2}$	0
Parallel Probe.	$\frac{N_0}{E_p}$	$\left(\frac{2}{3} \pi^2 C_2^2 \right) \frac{\sigma_D}{B_N^2}$	$\frac{E_b}{E_p} \beta_i$
Decision-Directed	$\frac{N_0}{E_p}$	$\left(\frac{2}{3} \pi^2 C_2^2 \right) \frac{\sigma_D}{B_N^2}$	$4E_b \bar{P}_e \beta_i$

β_i = average received power in the i th tap gain
 N_0 = variance of the thermal noise
 σ_D = rms Doppler spread
 $\bar{P}_e$ = average probability of error
 B_N = normalized bandwidth of the low-pass filter
 E_p = average received energy per probe bit
 E_b = average received energy per information bit.

where β_k represents the average power received in the k th tap gain.

In the decision-directed approach, the reference signal is the detected signal. If we define the error sequence as $e_n = \hat{a}_n - a_n$, where $\hat{a}_n$ is the detected symbol, the reference signal is given by $u_n = a_n + e_n$. Then the input to the low-pass filter is

$$\frac{1}{E_p} \{r_{n-k} u_n\} = \frac{|a_n|^2}{E_p} c_k(n) + e_n \frac{a_n}{E_p} c_k(n), \quad (16)$$

where $E_p = E_b$. The second term on the right-hand side is a source of noise associated with the reference-directed measurements. We refer to this noise as the internal measurement noise for the decision-directed approach.

For $a_n = \pm \sqrt{E_b}$, we observe that e_n can be either $\pm 2\sqrt{E_b}$ or zero. It is zero when the detected symbol is the same as the transmitted symbol; it is $+2\sqrt{E_b}$ when $a_n = -\sqrt{E_b}$ is transmitted and $\hat{a}_n = +\sqrt{E_b}$ is received; and it is $-2\sqrt{E_b}$ when $a_n = +\sqrt{E_b}$ is transmitted and $\hat{a}_n = -\sqrt{E_b}$ is received. Therefore, the random variable $e_n a_n$ can take only two values $-2E_b$ and zero with the following probabilities:

$$\begin{aligned} P\{e_n a_n = -2E_b\} &= \bar{P}_e \\ P\{e_n a_n = 0\} &= 1 - \bar{P}_e \end{aligned} \quad (17)$$

where $\bar{P}_e$ is the average probability of error.

Given the distribution function of these probabilities, one can determine the statistics of the internal measurement noise in the decision-directed method given in (16):

$$E \left\{ \frac{e_n a_n}{E_p} c_k(n) \right\} = -2\bar{P}_e c_k(n)$$

$$\sigma_I^2 = \text{Var} \left\{ \frac{e_n a_n}{E_p} c_k(n) \right\} = 4E_b \bar{P}_e |c_k(n)|^2 = 4E_b \bar{P}_e \beta_k. \quad (18)$$

The overall measurement noise at the input to the filters can now be evaluated by

$$\sigma_v^2 = \sigma_T^2 + \sigma_{\Delta v}^2 + \sigma_I^2. \quad (19)$$

Table I shows the variance of different measurement noises for different channel measurement techniques at the input to the low-pass filters. The measurement noise at the output of the channel estimator is found from (12).

V. PERFORMANCE PREDICTION FOR ADAPTIVE MATCHED FILTER

In this section we resort to vector notation to simplify the calculations. In vector notations, (4b) can be written as

$$R = aC + \eta$$

where a is the transmitted symbol, R is the vector of the received samples with length $L + 1$, C is the vector of tap gains, and η is the vector of white Gaussian noise. The covariance matrix of the noise is IN_o , with I the identity matrix and $N_o = E\{|\eta_i|^2\}$.

In the ideal case with perfect channel measurement, the decision for detection is made on the output of the discrete matched filter with tap gains C^* , where "*" stands for complex conjugate transpose vector. This output is given by the following equation:

$$RC^* = a|C|^2 + \eta C^*.$$

Define the instantaneous signal-to-noise ratio, ρ , by the square of the first term to variance of the second term of the above equation,

$$\rho = \frac{|a|^2 |C|^2}{E\{|\eta_i|^2\}} = \frac{E_b}{N_o} |C|^2,$$

and the average signal-to-noise ratio Γ as

$$E\{\rho\} = \Gamma = \frac{E_b}{N_o}.$$

As shown in [16] and [20], the average probability of error in terms of Γ is given by

$$\bar{P}_e = \sum_{n=0}^L \frac{\alpha_n}{2} \left[1 - \left[\frac{2\Gamma\lambda_n}{1 + 2\Gamma\lambda_n} \right]^{1/2} \right] \quad (20)$$

where λ_n 's are eigenvalues of the covariance matrix of tap gains $\Lambda = E\{CC^*\}$, and α_n 's are defined as

$$\alpha_n = \prod_{k=0, k \neq n}^L \frac{1}{1 - \lambda_k/\lambda_n}.$$

In terms of the parameters of the model, the elements of the covariance matrix Λ are given by

$$E\{c_n c_m^*\} = \int f \left(\frac{n}{2f_0} - u \right) Q(u) f^* \left(\frac{m}{2f_0} - u \right) du.$$

The noisy channel impulse response estimate used for adaptive matched filtering is $\hat{C}^* = C^* + W^*$, where W^* is the vector of measurement noises defined by Table I and (12). In this case, the output of the matched filter is given by

$$R\hat{C}^* = RC^* + RW^*.$$

The term RW^* is the contribution of measurement noises in detection process. Considering this new noise term, the average signal-to-noise ratio Γ' is given by

$$\begin{aligned} \Gamma' &= \frac{E_b}{N_o + E\{|RW^*|^2\}} = \frac{E_b}{N_o + \sum_{i=0}^L E\{|r_{n-i} w_i^*|^2\}} \\ &= \frac{\Gamma}{1 + \Gamma \sum_{i=0}^L \beta_i \sigma_{w_i}^2 + \sum_{i=0}^L \sigma_{w_i}^2}. \end{aligned}$$

By substituting Γ' , rather than Γ , in (20), one can determine the average probability of error in presence of measurement noise. In these calculations, the measurement noise is treated as an equivalent Gaussian noise averaged over all fades.

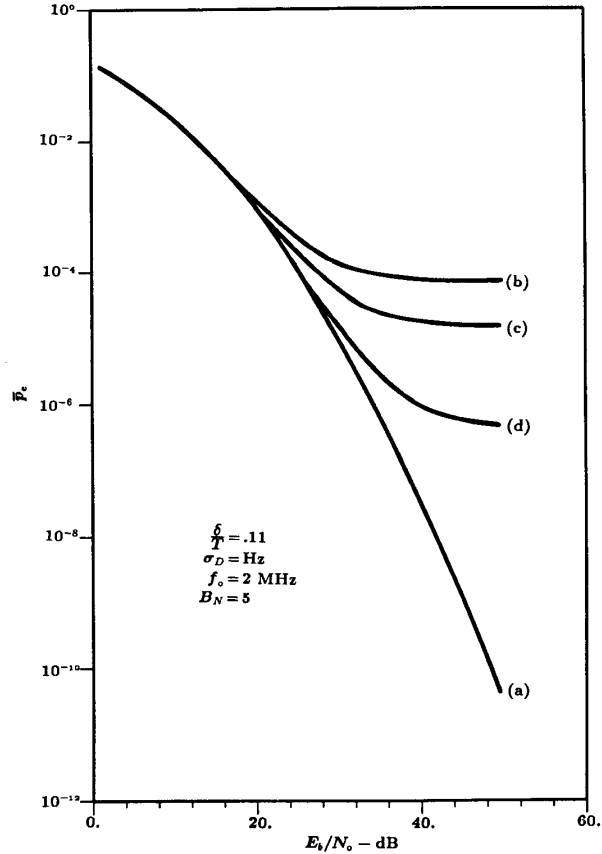


Fig. 5. Performance of various implementations of the discrete matched filter receiver over troposcatter channel: (a) is for ideal channel measurement and represents the upper bound of the theoretical performance; (b) is for the time multiplexed (serial) probe; (c) is for the nonmultiplexed (parallel) probe; and (d) is for the decision-directed method. The ratio of rms multipath spread to symbol time is 0.11; the rms Doppler spread is 10 Hz; the transmission bandwidth is 2 MHz; and the normalized bandwidth is 5.

VI. NUMERICAL RESULTS AND DISCUSSIONS

The system considered for performance prediction is one using a 100% cosine pulse with a bandwidth of 2 MHz. The samples at the receiver are taken at the Nyquist rate (4 MHz), and the rms Doppler spread is assumed to be 10 Hz.

For a ratio of multipath spread to baud interval of $\delta/T = 0.11$, the equivalent tapped delay line model for this example contains four significant tap gains. To avoid ISI, we associate an appropriate duty cycle with the waveform. The average received power for each tap gain and the eigenvalues of the covariance matrix of the tap gains are given in [16]. The average energy for the probe bit is assumed to be 10 dB below the average energy for an information bit (or one probe bit per each ten data bits with a time-multiplexed probe). Using these values, Table I, (12) and (20), curves of average probability of error versus E_b/N_o are generated and are shown in Figs. 5, 6, and 7 for normalized bandwidths of 5, 50, and 500, respectively.

As shown in the plots, the performance of the system saturates as the average energy per bit is increased with respect to the background noise. Measurement noises in Table I which are independent of the thermal noise will cause this condition.

With the decision-directed and serial probe, the only measurement noise independent of the thermal noise is the tap gain noise. Since the tap gain noise is inversely proportional to the square of the normalized bandwidth, the overall effect determined by Table I and

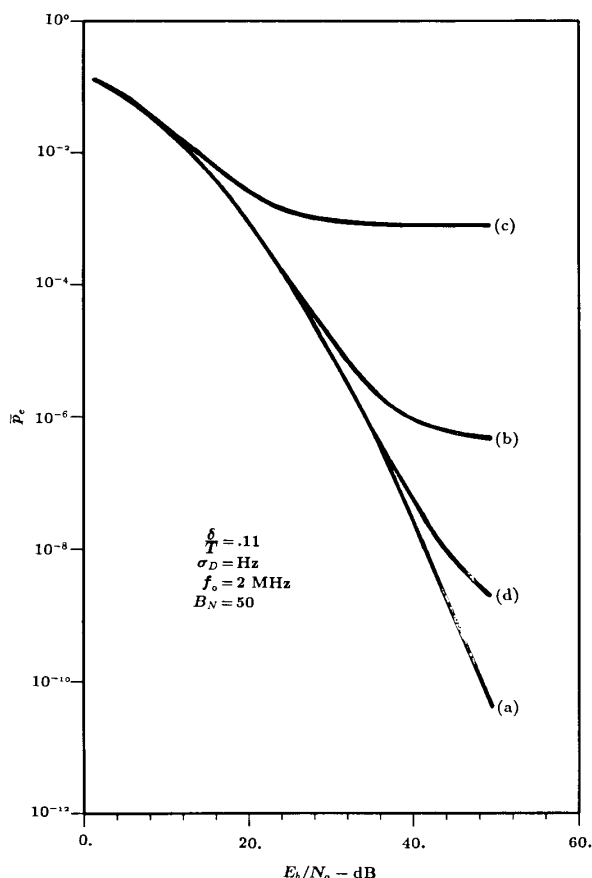


Fig. 6. Performance of various implementations of the discrete matched filter receiver over troposcatter channel: (a) is for ideal channel measurement and represents the upper bound of the theoretical performance; (b) is for the time multiplexed (serial) probe; (c) is for the nonmultiplexed (parallel) probe; and (d) is for the decision-directed method. The ratio of rms multipath spread to symbol time is 0.11; the rms Doppler spread is 10 Hz; the transmission bandwidth is 2 MHz; and the normalized bandwidth is 50.

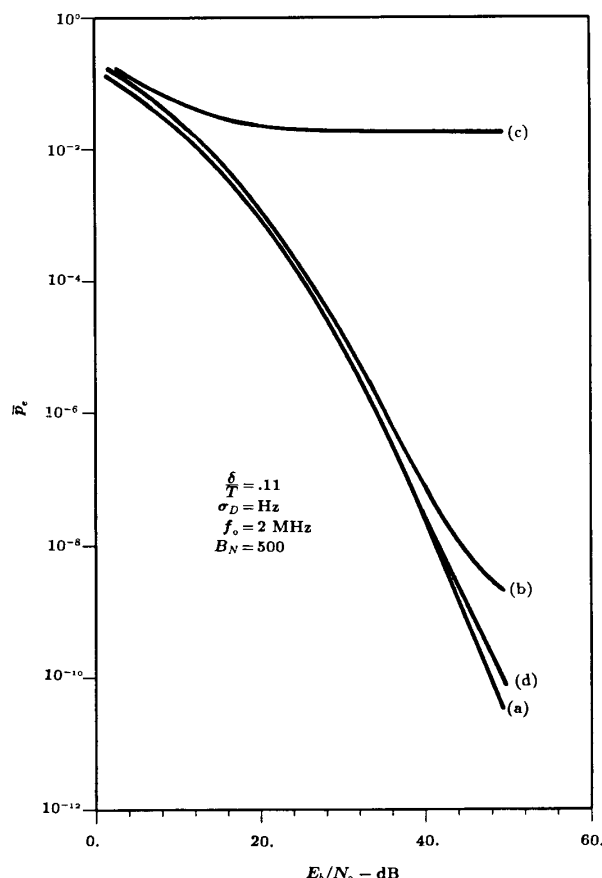


Fig. 7. Performance of various implementations of the discrete matched filter receiver over troposcatter channel: (a) is for ideal channel measurement and represents the upper bound of the theoretical performance; (b) is for the time multiplexed (serial) probe; (c) is for the nonmultiplexed (parallel) probe; and (d) is for the decision-directed method. The ratio of rms multipath spread to symbol time is 0.11; the rms Doppler spread is 10 Hz; the transmission bandwidth is 2 MHz; and the normalized bandwidth is 500.

(12) will be a noise term inversely proportional to B_N . Thus, the increase in normalized bandwidth results in saturation at higher values of SNR. This phenomenon is most severe with the serial probe, because the measurement time is α times longer than with the parallel or decision-directed probe.

With the parallel probe technique, both the tap gain variation noise and the internal measurement noise are independent of thermal noise. However, the internal measurement noise dominates and will generally do so with parameters similar to those in this example. (This is not the case for the decision-directed probe, where the internal noise will generally be much smaller than the tap gain noise.) The effect of (12) on the internal noise for the parallel probe is to produce an output noise term proportional to B_N . Thus, higher normalized bandwidths result in higher measurement noise which means saturation at lower values of SNR.

In deep fades the probability of error is very high, and when an error occurs the tap gain estimations are degraded. This degradation increases the probability of an erroneous decision on the next digit. The phenomenon may stop proper operation of the system after deep fades. However, as observed by simulation in [19] and shown analytically in [16] it does not occur for QPSK. In fact, for QPSK communication, channel estimation recovers even after the deep fades. The only penalty is a $\pm 90^\circ$ phase ambiguity [16] which can

be resolved, although at some cost in SNR [18], by differentially encoding both inphase and quadrature phase data sequences.

VII. CONCLUSION

A unified discrete channel model from the information source up to the sampler was developed for fading multipath channels. Different methods for adaptive channel measurement were studied. The performance of a discrete matched filter using different adaptation techniques and working over a troposcatter channel was predicted. Within the context of the model and the assumption presented in this paper, the effects of channel measurement noise are less damaging with the decision-directed adaptation technique than with a transmit reference probe. The superiority over the serial probe can be traced to the tap gain update every baud with the decision-directed approach, and the main disadvantage of the parallel probe is the presence of the much larger information signal in the channel estimation circuitry.

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